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PROPAGATION OF STRESS SHOCK WAVES IN ELASTOPLASTIC CONTINUOUS MEDIA WITH SPHERICAL SYMMETRY

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Abstract. *The article examines the motion caused by the explosion of a spherical charge in elastoplastic continuous media. When a charge explodes, it is assumed that the charge instantly turns into a high-pressure gas without changing its volume, and this gas spreads into the environment, forming a shock wave with spherical symmetry. Taking into account the presence of tangential stresses in inclined areas, equations of one-dimensional dynamic motion of soil are derived, according to which the patterns of propagation of shock waves in soil massifs during an explosion with spherical symmetry are studied. To find the limiting state of the soil, the Prandtl plasticity condition was used, and also when determining the radial stress at the shock wave front, experimental dependencies between* $\sigma(\varepsilon^*)$ *and* $\sigma_i(\varepsilon^*, \varepsilon_i^*)$ *were used. The general solution of the problem and the necessary values for its numerical solution in the case of constant density on the shock wave are given.*

Keywords. *Shock wave, explosion, normal and shear stress, spherical symmetry, medium density, intensity, plastic gas.*

Annotatsiya: *Maqolada elastik-plastik tutash muhitlarda sferik zaryadning portlashi natijasida yuzaga keladigan harakat ko‘rib chiqilgan. Zaryad portlaganda bir lahzada, hajmini o‘zgartirmasdan, yuqori bosimli gazga aylanadi va bu gaz atrof-muhitga sferik simmetriyali zarba to‘lqinining shakllanishi bilan tarqaladi, deb taxmin qilinadi. Bunda qiya yuzalarda urinma kuchlanishlar mavjudligini hisobga olgan holda, muhitning bir o‘lchovli dinamik harakati tenglamalari olingan va unga ko‘ra sferik simmetriyali portlash paytida grunt massivlarida zarba to‘lqinlarining tarqalish qonuniyatlari o‘rganilgan. Gruntning chegaraviy holatini topish uchun Prandtlyaning plastiklik shartidan, shuningdek zarba to‘lqini frontidagi radial kuchlanishni aniqlashda* $\sigma(\varepsilon^*)$ *va* $\sigma_i(\varepsilon^*, \varepsilon_i^*)$ *lar orasidagi eksperimental bog‘liqliklardan ham foydalanilgan. Masalaning umumiy yechimi va uni sonli yechish uchun zarur qiymatlar zarbiy to‘lqindagi zichlik o‘zgarmas bo‘lgan holda keltirilgan (98 so‘z).*

Tayanch so‘zlar: *Zarbiy to‘lqin, portlash, normal va urinma kuchlanish, sferik simmetriya, muhit zichligi, intensivlik, plastik gaz.*

Аннотация. *В статье рассмотрено движение, вызванное взрывом сферического заряда в упругопластических сплошных средах. При взрыве заряда предполагается, что заряд мгновенно превращается в газ высокого давления, не изменяя своего объема, и этот газ распространяется в окружающую среду, образуя ударную волну со сферической симметрией. Учитывая наличие касательных напряжений в наклонных площадках выведены уравнения одномерного динамического движения грунта, согласно которому исследуются закономерности распространения ударных волн в грунтовых массивах при взрыве со сферической симметрией. Для нахождения предельного состояния грунта использовано условие пластичности Прандтля, а также при определении радиального напряжения на фронте ударной волны использованы экспериментальные зависимости между* $\sigma(\varepsilon^*)$ *и* $\sigma_i(\varepsilon^*, \varepsilon_i^*)$. *Приведены общее решение задачи и необходимые значения для ее численного решения в случае постоянной плотности на ударной волне.*

Ключевые слова*. Ударная волна, взрыв, нормальное и касательное напряжения, сферическая симметрия, плотность среды, интенсивность, пластический газ.*

Introduction

Integrated design issues, i.e. calculation and design, construction, reconstruction and restoration of tunnels and underground structures (service and technological premises, garages, warehouses, multi-level underground parking lots, underground

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shopping complex, transport interchanges and other large and complex structures) are quite problematic, since they require ensuring the complete safety of existing engineering communications. In this case, the main emphasis is on the applied significance of the methodology for the integrated design of tunnels and underground structures as a fragment of a single process of development of underground space.

The methodology of integrated design also includes other questions about how to calculate the support of underground mine workings, how and on what principles to choose methods of construction of objects, how to carry out pile foundation construction, injection strengthening of soils, installation of supports in water areas (at depths exceeding hundreds of meters, including supports for bridges across sea straits, supports for the installation of drilling platforms, etc.) and how to assess the basic qualities and technical and economic feasibility of constructing tunnels and underground structures.

For a joint decision on the choice of technological, architectural, structural and space-planning solutions for the construction of critical objects, taking into account the properties of “plasticity” (the ability to undergo irreversible deformations) and creep, relaxation (changes in mechanical properties and state over time) of soil and rock massifs, it is also necessary to experimentally theoretical studies of the problems of shock wave propagation at high stresses and high strain rates [1-7, 10, 13, 14, 20, 21].

This article will outline the theory of one-dimensional dynamic motion of a soil mass having elastoplastic properties during an explosion with spherical symmetry.

Derivation of the Equation Taking into Account Tangential Stresses

Let us select an infinitesimal element from a moving medium - soil, which occupies the entire space during an explosion of an explosive with two pairs of mutually perpendicular meridional sections and two concentric spherical surfaces. Normal and tangential stress are denoted through σ and τ with the corresponding index according to the standard in mechanics.

Let us find out the general nature of the stressed state of an elementary element, the faces of which are the main areas and the main normal stresses act on them, respectively $\sigma_r \geq \sigma_\psi \geq \sigma_\varphi$.

Taking into account the invariance of the main stresses acting on the main platforms, after appropriate transformations we obtain the basic formula for motion during an explosion with spherical symmetry, taking into account tangential stresses in inclined areas [7]:

$$\rho_0 r^2 \cdot \frac{\partial^2 u}{\partial t^2} = (r+u)^2 \cdot \frac{\partial \sigma_r}{\partial r} + (\sigma_r - \sigma_\varphi) \cdot \frac{\partial}{\partial r}(r+u)^2 + \frac{\partial \tau_{r\varphi}}{\partial \varphi} \frac{\partial}{\partial r}(r+u)^2 \quad (1)$$

where, r - distance from the origin of coordinates to the occurrence of motion, u - displacement, t - time, ρ_0 - initial mass density.

According to the circle of Otto Christian More [8, 9]:

$$\tau_{r\varphi} = \frac{\sigma_1 - \sigma_2}{2} \sin 2\varphi, \ 0 \leq \varphi \leq 180^\circ \quad 2)$$

here, σ_1, σ_2 - main stresses, which are determined by the formulas:

$$\sigma_1 = \frac{1}{2}\left[\sigma_r + \sigma_\varphi + \sqrt{(\sigma_r - \sigma_\varphi)^2 + 4\tau_{r\varphi}^2}\right], \sigma_2 = \frac{1}{2}\left[\sigma_r + \sigma_\varphi - \sqrt{(\sigma_r - \sigma_\varphi)^2 + 4\tau_{r\varphi}^2}\right] \quad (3)$$

We substitute these relations in (2) and find:

$$\tau_{r\varphi} = \frac{\sigma_r - \sigma_\varphi}{2} \cdot tg 2\varphi \quad (4)$$

Let us consider the Prandtlya plasticity condition, or, what is the same, the limiting state condition of the soil, which is written in the form:

$$\frac{\sigma_1 - \sigma_3}{2} = k \cos v - \frac{\sigma_1 + \sigma_2}{2} \sin v$$

where, k - adhesion coefficient, v - angle of internal friction, $\sigma_1 \geq \sigma_2 \geq \sigma_3$ - main normal stresses (mn). When designating $-\tau_0 = k \cos v, \ \mu = \sin v$ and $\sigma_1 = \sigma_r^{mn}, \ \sigma_2 = \sigma_3 = \sigma_\varphi^{mn} = \sigma_\psi^{mn}$ plasticity conditions have the form:

$$\sigma_r^{mn} - \sigma_\varphi^{mn} = -\tau_0 + \mu\left(\sigma_r^{mn} + \sigma_\varphi^{mn}\right) \quad (3)$$

If we introduce the notation $\sigma_r^{mn} \geq \sigma_\varphi^{mn} = \sigma_\psi^{mn}$, which are also main stresses, formula (3) takes the form:

$$\sigma_r^{mn} = \frac{1}{2}\left[\sigma_r + \sigma_\varphi + \sqrt{(\sigma_r - \sigma_\varphi)^2 + 4\tau_{r\varphi}^2}\right], \ \sigma_\varphi^{mn} = \frac{1}{2}\left[\sigma_r + \sigma_\varphi - \sqrt{(\sigma_r - \sigma_\varphi)^2 + 4\tau_{r\varphi}^2}\right]$$

Using this equality, from (4) bearing in mind (4) we obtain:

$$\sigma_r - \sigma_\varphi = -\frac{\tau_0 \cos 2\varphi}{1+\mu\cos 2\varphi} + \frac{2\mu\cos 2\varphi}{1+\mu\cos 2\varphi}\sigma_r \quad (6)$$

(6) we substitute into (4) and take the partial derivative with respect to φ:

$$\frac{\partial \tau_{r\varphi}}{\partial \varphi} = \tau_0 \cdot I(\varphi) - 2\mu \cdot I(\varphi)\sigma_r$$

where,

$$I(\varphi) = \frac{\sin^2 2\varphi}{\cos 2\varphi(1+\mu\cos 2\varphi)^2} - \frac{1}{\cos 2\varphi(1+\mu\cos 2\varphi)}$$

.

The last relation of the partial derivative and (6), substituting into formula (1) we obtain:

$$(r+u)^2\frac{\partial \sigma_r}{\partial r} + 2\mu\left[\frac{\cos 2\varphi}{1+\mu\cos 2\varphi} - I(\varphi)\right]\frac{\partial}{\partial r}(r+u)^2\sigma_r = \rho_0 r^2\frac{\partial^2 u}{\partial t^2} + \tau_0\left[\frac{\cos 2\varphi}{1+\mu\cos 2\varphi} - I(\varphi)\right]\frac{\partial}{\partial r}(r+u)^2$$

Let us introduce the notation:

$$\nu(\varphi) = 4\mu\left[\frac{\cos 2\varphi}{1+\mu\cos 2\varphi} - I(\varphi)\right],\ A(\varphi) = \frac{\cos 2\varphi}{1+\mu\cos 2\varphi} - I(\varphi) = \frac{\nu(\varphi)}{4\mu}$$

Then the latter takes the form:

$$(r+u)^2\frac{\partial \sigma_r}{\partial r} + \frac{\nu(\varphi)}{2}\frac{\partial}{\partial r}(r+u)^2\cdot\sigma_r = \rho_0 r^2\frac{\partial^2 u}{\partial t^2} + \tau_0 A(\varphi)\frac{\partial}{\partial r}(r+u)^2 \quad (7)$$

Where, the mechanical parameter $\nu(\varphi)$ included in equations (7) shows that the presence of internal friction in the soil leads to faster braking of the movement caused by the explosion.

Now we use the basic relations for a shock wave, which represent a discontinuity surface moving in a medium, upon passing through which all its physical parameters change abruptly. The surface of the rupture is called the shock wave front.

First, we will use the law of conservation of mass of a selected particle on a shock wave:

$$\frac{1}{3}\cdot\frac{\partial}{\partial r}(r+u)^3 = \frac{\rho_0}{\rho}\cdot r^2\tau \quad (8)$$

System of equations (1), (5) and (8) contains the following unknown functions: main stresses σ_r and σ_φ; tangential stresses $\tau_{r\varphi}$ (the law of pairing of tangential stresses is taken into account); displacement u and density ρ, and therefore, it is not closed.

To close the system of equations, it is necessary to accept as an additional connection the relationship between $\sigma(\varepsilon)$ and $\sigma_i(\varepsilon,\varepsilon_i)$ at high dynamic stresses and high strain rates, for the case of loading and unloading of soils or rocks [20, 21].

Mechanical Formulation and Solution Method

We consider the movement caused by the explosion of a charge in a homogeneous soil occupying the entire space, i.e., without taking into account the boundary surfaces. When a spherical charge of radius r_0 explodes in the ground, it is assumed that at some moment, instantly, without changing volume, the charge turns into a high-pressure gas. This gas expands according to the polytropic law with the polytropic index γ.

It is assumed that the resulting high pressure spreads throughout the environment with the formation of a shock wave.

It is required to find the resulting motion of the medium.

Soil is considered as a plastic gas. Consequently, during unloading, the density previously acquired by the particle does not change. Based on the properties of plastic gas, it follows that in the region of ground motion behind the shock wave, the density is a function of the Lagrange coordinate r and does not depend on time.

Let us turn to the tense state of the environment.

Based on the results of experimental studies, the experimental relationship between hydrostatic pressure and medium density [2] takes the form:

$$\frac{\sigma_r + 2\sigma_\varphi}{3} = F\left(1 - \frac{\rho_0}{\rho}\right) \quad (9)$$

Movements with spherical symmetry, conditions (9) and plasticity conditions are sufficient to determine stress as a function of density

$$\sigma_r = \psi\left(1 - \frac{\rho_0}{\rho}\right)$$

Now let's move on to solving the problem.

Taking into account the above, to solve (1) we multiply both sides of equation (7) by $(r+u)^{\nu-2}$, integrate over r and, provided that $r_0 + u(r_0,t) = R(t)$ and $\sigma_r(r_0,t) = \sigma_0(t)$, we obtain an integro-differential equation:

$$\sigma_r(r,t)(r+u)^\nu = \rho_0\int_{r_0}^{r}\frac{r^2}{(r+u)^{2-\nu}}\cdot\frac{\partial^2 u}{\partial t^2}dr + \frac{2\tau_0 A}{\nu}\left[(r+u)^\nu - R^\nu\right] + R^\nu\sigma_0(t) \quad (10)$$

Due to the symmetry of the problem, we restrict ourselves to studying the propagation of the wave to the right. In this case we have the following initial and boundary conditions:

$$r \succ r_0,\ t=0,\ u_r = u_t = 0,\ r = r_0,\ t \succ 0,\ -\sigma_r = p_f(t)$$

where, $p_f(t)$ - given time function.

Expression (10) on the shock wave will take the form

$$\sigma_r^{\ *} r^{*\nu} = \rho_0 \int_{r_0}^{r^*} \frac{r^2}{(r+u)^{2-\nu}} \cdot \frac{\partial^2 u}{\partial t^2} dr + \frac{2\tau_0 A(\varphi)}{\nu}\left[r^{*\nu} - R^\nu\right] + R^\nu \sigma_0(t)$$

Considering the latter and (10) together gives

$$\sigma_r (r+u)^\nu - \sigma_r^{\ *} r^{*\nu} = -\rho_0 \int_{r_0}^{r} \frac{r^2}{(r+u)^{2-\nu}} \cdot \frac{\partial^2 u}{\partial t^2} dr - \frac{2\tau_0 A(\varphi)}{\nu}\left[r^{*\nu} - (r+u)^\nu\right]$$

The soil continuity equation in Lagrange variables in the case of spatial motion of cohesive soils, the application of which was proposed by academician X.A.Rakhmatulin, has the form:

$$\frac{1}{3} \cdot \frac{\partial}{\partial r}(r+u)^3 = \frac{\rho_0}{\rho(r)} \cdot r^2$$

Let's integrate this equation over r and get:

$$(r+u)^3 = 3\psi(r) + R^3(t) \tag{11}$$

where

$$\psi(r) = \int_{r_0}^{r} \frac{\rho_0 r^2}{\rho(r)} dr \tag{12}$$

From (11) by differentiating for speed and acceleration we obtain:

$$\frac{\partial u}{\partial t} = \frac{R^2(t)\cdot \dot R(t)}{\left[3\psi(r) + R^3(t)\right]^{2/3}},\ \frac{\partial^2 u}{\partial t^2} = \frac{2R(t)\cdot \dot R^2(t) + R^2(t)\cdot \ddot R(t)}{\left[3\psi(r) + R^3(t)\right]^{2/3}} - \frac{2\left[R^2(t)\cdot \dot R(t)\right]^2}{\left[3\psi(r) + R^3(t)\right]^{5/3}} \tag{13}$$

For the “shock wave” coordinate r^*, formulas (11) and (12) give:

$$r^{*3} = 3\psi(r) + R^3(t) = 3\int_{r_0}^{r^*} \frac{\rho_0 r^2}{\rho(r)} dr + R^3(t) \tag{14}$$

Now let us write down the dynamic conditions at the shock wave front, resulting from the laws of conservation of mass and momentum:

$$u_t^* + D\cdot u_r^* = u_t^{**} + D\cdot u_r^{**},\ \rho^* D u_t^*\left(1+u_r^*\right) + \sigma_{r(d)}^* = \rho^{**} D u_t^{**}\left(1+u_r^{**}\right) + \sigma_{r(g)}^{**}$$

where, $\sigma_{r(d)}^* = \overline{\sigma_r^*} + \overline{\overline{\sigma_r^*}}$ - stress corresponding to the disturbed part of the medium; $\overline{\sigma_r^*} = \sigma_r^*$ - shock wave front stress; $\overline{\overline{\sigma_r^*}} = \lambda\left(\varepsilon^*, \varepsilon_i^*\right)\cdot \varepsilon^* + 2G\left(\varepsilon^*, \varepsilon_i^*\right)\cdot \varepsilon_r^*$ - stress corresponding to the “disturbed part” of the medium; $\sigma_{r(g)}^{**} = \overline{\sigma_r^{**}} + \overline{\overline{\sigma_r^{**}}}$ - stress corresponding to the undisturbed part of the medium; $\overline{\sigma_r^{**}} = -p_\alpha$ - stress equal to atmospheric pressure, $\overline{\overline{\sigma_r^{**}}} = 0$ since in the “rest” region the soil is in its natural state.

Then the dynamic conditions on the shock wave take the form [7]:

$$-\sigma_r^{\ *} = \frac{\rho_0 \dot u^{*2}}{1-b(r^*)} + \left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*} + \frac{4}{9}\cdot\frac{\sigma_i\left(\varepsilon^*, \varepsilon_i^*\right)}{\varepsilon_i^{\ *}}\right]\cdot \varepsilon_i^{\ *} + p_\alpha,\ D = \frac{\dot u^*}{1-b(r^*)}$$
$$b(r^*) = \frac{\rho_0}{\rho(r^*)} = \left.\left(\frac{d\psi(r)}{dr}\cdot\frac{1}{r^2}\right)\right|_{r=r^*},\ R(t) = \sqrt[3]{r^{*3} - 3\int_{r_0}^{r^*} \frac{\rho_0}{\rho(r)} r^2 dr} \tag{15}$$

where, $\rho(r^*)$ - medium density, r^* - shock wave coordinates, D - shock wave speed, u_t^* - particle velocity on the shock wave, $R(t)$ - cavity radius.

According to (12) and (14), the velocity of a medium particle on a shock wave is:

$$\dot u^* = \frac{R^2(t)\cdot \dot R(t)}{r^{*2}}$$

Accordingly, we rewrite the conditions for voltage (15):

$$-\sigma_r^{\ *} = \frac{\rho_0}{1-b}\cdot\frac{\left(R^2\cdot \dot R\right)^2}{r^{*4}} + \left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*} + \frac{4}{9}\cdot\frac{\sigma_i\left(\varepsilon^*, \varepsilon_i^*\right)}{\varepsilon_i^{\ *}}\right]\cdot \varepsilon_i^{\ *} + p_\alpha,\ D = \frac{R^2\cdot \dot R}{(1-b)r^{*2}}$$

Substituting the values of acceleration (13) and voltage into (9), we obtain:

$$-(r+u)^{\nu}\sigma_r=\left(R^2\ddot{R}+2R\dot{R}^2\right)\rho_0\int_r^{r^*}\frac{r^2dr}{\left(3\psi+R^3\right)^{\frac{4-\nu}{3}}}-2\left(R^2\dot{R}\right)^2\rho_0\int_r^{r^*}\frac{r^2dr}{\left(3\psi+R^3\right)^{\frac{7-\nu}{3}}}+$$
$$+\frac{\rho_0}{1-b}\frac{\left(R^2\dot{R}\right)^2}{r^{*4-\nu}}+\frac{2\tau_0 A}{\nu}\left(r^{*\nu}-(r+u)^{\nu}\right)+p_\alpha r^{*\nu}+\left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*}+\frac{4}{9}\cdot\frac{\sigma_i(\varepsilon^*,\varepsilon_i^*)}{\varepsilon_i^*}\right]\cdot\varepsilon_i^* r^{*\nu} \tag{16}$$

In these equations, for the sake of simplicity of notation, dependence (11) is used only in the integrands.

From equation (16) it follows that the motion parameters are determined if the functions are known: $\psi(r)$ and $R(t)$.

First, let's look at a particular case - the density on the shock wave is constant ($b(r)=const$):

$$\frac{\rho_0}{\rho(r)}=\frac{\rho_0}{\rho_1}=1-\varepsilon=b_1$$

According to (12) and (14) we will have

$$\psi(r)=\int_{r_0}^{r}b_1r^2dr=b_1\frac{r^3-r_0^{\,3}}{3},\ r^{*3}=3b_1\int_{r_0}^{r^*}r^2dr+R^3,\ r^{*3}=\frac{R^3-b_1r_0^{\,3}}{1-b_1} \tag{17}$$

Substituting the value $\psi(r)$ into equations (16) and under the boundary conditions - $(r+u)|_{r=r_0}=r_0+u(r_0,t)=R$, we have:

$$-\sigma_r(r_0,t)\cdot R^{\nu}=\left(R^2\ddot{R}+2R\dot{R}\right)^2\cdot\frac{\rho_0}{b_1(\nu-1)}\left(\frac{1}{r^{*1-\nu}}-\frac{1}{R^{1-\nu}}\right)-\frac{2\rho_0}{b_1(\nu-4)}\left(R^2\dot{R}\right)^2\left(\frac{1}{r^{*4-\nu}}-\frac{1}{R^{4-\nu}}\right)+$$
$$+\frac{\rho_0}{1-b_1}\frac{\left(R^2\dot{R}\right)^2}{r^{*4-\nu}}+\frac{2\tau_0A}{\nu}\left(r^{*\nu}-R^{\nu}\right)+p_\alpha r^{*\nu}+\left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*}+\frac{4}{9}\cdot\frac{\sigma_i(\varepsilon^*,\varepsilon_i^*)}{\varepsilon_i^*}\right]\cdot\varepsilon_i^*r^{*\nu} \tag{18}$$

We will assume that the pressure in the cavity is given and for the polytropic law of cavity expansion we obtain [11, 12]:

$$p_k(R)=p_0\left(\frac{r_0}{R}\right)^{2\gamma}$$

where, p_0 - gas pressure at the moment of instantaneous explosion. Due to the boundary condition $-\sigma_r(r_0,t)$, it can be replaced by pressure p_k, then we perform the following replacement

$$y(R)=\dot{R}^2,\quad y'(R)=\frac{dy(R)}{dR}=2\ddot{R}$$

equation (18) can be reduced to the equation:

$$\frac{R^2}{\nu-1}\left(\frac{1}{r^{*1-\nu}}-\frac{1}{R^{1-\nu}}\right)y'+y\left[\frac{4R}{\nu-1}\left(\frac{1}{r^{*1-\nu}}-\frac{1}{R^{1-\nu}}\right)-\frac{4R^4}{\nu-4}\left(\frac{1}{r^{*4-\nu}}-\frac{1}{R^{4-\nu}}\right)+\frac{2b_1}{1-b_1}\frac{R^4}{r^{*4-\nu}}\right]=$$
$$=\left\{p_0\left(\frac{r_0}{R}\right)^{3\gamma}R^{\nu}-p_\alpha r^{*\nu}-\frac{2\tau_0A}{\nu}\left(r^{*\nu}-R^{\nu}\right)-\left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*}+\frac{4}{9}\cdot\frac{\sigma_i(\varepsilon^*,\varepsilon_i^*)}{\varepsilon_i^*}\right]\cdot\varepsilon_i^*r^{*\nu}\right\}\cdot\frac{2b_1}{\rho_0} \tag{19}$$

Obviously, (19) can be reduced to the form

$$y'+F(R)\cdot y=Q(R)$$

The solution of which is given by the formula

$$y=e^{-\int_{r_0}^{R}F(R)dR}\cdot\left\{y_0+\int_{r_0}^{R}Q(R)\cdot e^{\int_{r_0}^{R}F(R)dR}dR\right\} \tag{20}$$

where

$$F(R)=\left\{\frac{4R}{\nu-1}\left(\frac{1}{r^{*1-\nu}}-\frac{1}{R^{1-\nu}}\right)-\frac{4R^4}{\nu-4}\left(\frac{1}{r^{*4-\nu}}-\frac{1}{R^{4-\nu}}\right)+\frac{2b_1}{1-b_1}\frac{R^4}{r^{*4-\nu}}\right\}\cdot\left[\frac{R^2}{\nu-1}\left(\frac{1}{r^{*1-\nu}}-\frac{1}{R^{1-\nu}}\right)\right]^{-1}$$

$$Q(R)=\left\{p_0\left(\frac{r_0}{R}\right)^{3\gamma}R^{\nu}-p_\alpha r^{*\nu}-\frac{2\tau_0A}{\nu}\left(r^{*\nu}-R^{\nu}\right)-\left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*}+\frac{4}{9}\cdot\frac{\sigma_i(\varepsilon^*,\varepsilon_i^*)}{\varepsilon_i^*}\right]\cdot\varepsilon_i^*r^{*\nu}\right\}\times$$
$$\times\frac{2b_1}{\rho_0}\cdot\left[\frac{R^2}{\nu-1}\left(\frac{1}{r^{*1-\nu}}-\frac{1}{R^{1-\nu}}\right)\right]^{-1}$$

From the coefficients of equations (19) it is clear that the integrals included in (20) in the lower limit give uncertainty. However, it can be shown that the integrals included in this solution, improper at the lower limit, tend to zero when the upper limit of integration tends to the lower.

When $t=0$, $R=r_0$, $r^*=r_0$ we get the initial value y_0:

$$y_0 = \frac{1-b_1}{\rho_0} \cdot \left\{ p_0 - p_\alpha - \left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*} + \frac{4}{9} \cdot \frac{\sigma_i(\varepsilon^*, \varepsilon_i^*)}{\varepsilon_i^*} \right] \cdot \varepsilon_i^* \right\}$$

Since the coefficient of the derivative for the initial data $(R = r = r_0)$ vanishes, to solve numerically (19) it is necessary to indicate a method for determining derivatives of the desired function.

To determine $y'(0)$, equation (19) (at $\nu \neq 0$) is differentiated by R and into the resulting expression we substitute the initial value $t = 0,\ R = r^* = r_0$

$$y_0' = \frac{8b_1}{r_0(1-3b_1)} y_0 - \left(p_0(\nu - 3\gamma) + 2\tau_0 A\right) \cdot \frac{2b_1}{\rho_0} \cdot \frac{1-b_1}{(1-3b_1)r_0}$$

Using this method, derivatives of all orders can be determined.

Conclusions

The main differences of this study are as follows:

1. Taking into account the effects of tangential stresses acting on the sliding area, the equations of one-dimensional dynamic motion of the soil are derived. In the particular case when there are no tangential stress components in inclined areas during an explosion of an explosive substance, we have a similar equation of dynamic motion obtained in [21].

2. When modeling soil as an elastoplastic medium, the results of experimental studies to determine the mechanical properties of soils at high stresses in the $\sigma(\varepsilon)$ form $\sigma_i(\varepsilon, \varepsilon_i)$ and were used, carried out by the author under the direct supervision of Academician X.A.Rakhmatulin.

3. When determining the radial stress at the shock wave front, experimental dependences $\sigma(\varepsilon^*)$ and $\sigma_i(\varepsilon^*, \varepsilon_i^*)$ were used.

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