Technical science and innovation

Volume 2024 | Issue 2 Article 9

5-13-2024

REGULAR ALGORYTHMS FOR SYNTHESIS OF OPTIMAL CONTROL SYSTEMS FOR DYNAMIC OBJECTS

Igamberdiyev Husan
Tashkent State Technical University, Tashkent city, Republic of Uzbekistan, DSc, professor, academician, ihz.tstu@gmail.com, https://orcid.org/0000-0003-3047-526;

Uktam Farkhodovich Mamirov
Tashkent State Technical University, Tashkent city, Republic of Uzbekistan, DSc, Associate professor, uktammamirov@gmail.com, https://orcid.org/0000-0002-5596-2819;, uktammamirov@gmail.com

Xankeldiyeva Zebiniso
Bukhara engineering - technological institute, Bukhara city, Republic of Uzbekistan, Assistant, bizi-84@mail.ru, https://orcid.org/0009-0005-2893-3704.

DOI: https://doi.org/10.59048/2181-0400
E-ISSN: 2181-1180
.1587

REGULAR ALGORYTHMS FOR SYNTHESIS OF OPTIMAL CONTROL SYSTEMS FOR DYNAMIC OBJECTS

KH.Z.IGAMBERDIEV[1], U.F. MAMIROV[1], Z.KH.KHANKELDIYEVA[2](1 – Tashkent State Technical University, Tashkent city; 2 – Bukhara engineering - technological institute, Bukhara city, Republic of Uzbekistan)*

Received: April 22, 2024; Accepted: May 13, 2024; Online: July 01, 2024.

Abstract: *The article presents algorithms for the synthesis of an optimal control system for dynamic objects. As a model, we consider a differential equation of a continuous one-dimensional system in the form of a state space, which has the properties of controllability and observability. The paper shows the need to use an observation device in order to assess how the properties of the controlled system change with the slightest change in the parameters of the control object, to assess the sensitivity of the system to these changes. When finding a solution to the equation formulated to find the parameters of the control law, computational difficulties arise due to the fact that the system of equations is, as a rule, ill-conditioned. Considering the ill-posedness of the problem under consideration, regular procedures were used. The above algorithms make it possible to synthesize a stable control system with an optimal feedback gain.*

Key words: *dynamic object, optimal control system, synthesis algorithms, ill-conditioned system, regular procedures, stable control system, optimal feedback gain.*

Annotatsiya: *Maqolada dinamik ob'yektlarni boshqarishning optimal tizimini sintez qilish algoritmlari keltirilgan. Model sifatida boshqariluvchanlik va kuzatiluvchanlik xususiyatlariga ega bg'lgan holat fazosi kg'rinishidagi uzluksiz bir g'lchovli tizimning differensial tenglamasi kg'rib chiqiladi. Ishda boshqariladigan tizimning xususiyatlari boshqaruv ob'ekti parametrlarining ozgina g'zgarishi bilan qanday g'zgarishini baholash, tizimning ushbu g'zgarishlarga sezgirligini baholash uchun kuzatuv moslamasidan foydalanish zarurligi kg'rsatilgan. Boshqarish qonunining parametrlarini topish uchun tuzilgan tenglamaning yechimini topishda tenglamalar tizimi, yomon shartlangan bg'lganligi sababli, uni hisoblashda qator qiyinchiliklar yuzaga keladi. Kg'rib chiqilayotgan masalaning nokorrekt masala ekanligini hisobga olib, muntazam protseduralardan foydalanildi. Keltirilgan algoritmlar teskari aloqaning optimal kuchaytirish koeffitsiyentiga ega turg'un boshqarish tizimini sintez qilish imkonini beradi.*

Kalit sg'zlar: *dinamik ob'yekt, optimal boshqarish tizimi, sintezlash algoritmlari, yomon shartlangan tizim, muntazam protseduralar, turg'un boshqarish tizimi, teskari aloqaning optimal kuchaytirish koeffitsiyenti.*

Аннотация: *В статье приводятся алгоритмы синтеза оптимальной системы управления динамическими объектами. В качестве модели рассматривается дифференциальное уравнение непрерывной одномерной системы в форме пространства состояния, обладающая свойствами управляемости и наблюдаемости. В работе показана необходимость использования устройства наблюдения с целью оценки того, как изменяются свойства управляемой системы при малейшем изменении параметров объекта управления, оценки чувствительности системы к этим изменениям. При отыскании решения уравнения, сформулированного для нахождения параметров закона управления, возникают трудности вычислительного характера, обусловленные тем обстоятельством, что система уравнений является, как правило, плохо обусловленной. Учитывая некорректность рассматриваемой задачи были использованы регулярные процедуры. Приведенные алгоритмы позволяют синтезировать устойчивую систему управления с оптимальным коэффициентом усиления обратной связи.*

Ключевые слова: *динамический объект, оптимальная система управления, алгоритмы синтеза, плохо обусловленная система, регулярные процедуры, устойчивая система управления, оптимальный коэффициент усиления обратной связи.*

Introduction

Ensuring a minimum root-mean-square quality criterion cannot be the only requirement for a control system. In order for the system to be operational, it must satisfy a set of requirements, among which the most important is the requirement of stability,

*Igamberdiyev Khusan Zakirovich – DSc, professor, academician, ihz.tstu@gmail.com, https://orcid.org/0000-0003-3047-526;
Mamirov Uktam Farxodovich – DSc, Associate professor, uktammamirov@gmail.com, https://orcid.org/0000-0002-5596-2819;
Khankeldiyeva Zebiniso Khabibovna – Assistant, bizi-84@mail.ru, https://orcid.org/0009-0005-2893-3704.

preservation of quality in case of deviations of parameters from the calculated values that are inevitable in practice. Therefore, it is very important to assess how the characteristics of the controlled system will change with small variations in parameters, to assess the sensitivity of the system to these variations [1-5].

Let us note right away that for optimal systems the variations in system characteristics will be less than for non-optimal systems (with the same changes in parameters). Indeed, optimal systems provide a minimum quality criterion, but at the minimum point the main, linear, term of the criterion increment with variations in parameters vanishes. At the minimum point, the increment of the criterion with variations in parameters will be determined only by terms of higher degrees, and, therefore, the sensitivity of the optimal system to variations of parameters will generally be small - less than the sensitivity of non-optimal systems.

Of course, this circumstance is very favorable for the design and operation of optimal systems. It is the low sensitivity of the quality criterion for optimal systems to variations in parameters that allows one to relatively boldly make simplifying assumptions when synthesizing optimal systems and not be afraid that deviations of actual parameters from calculated ones (unless these deviations are too large) significantly worsen the performance of the system. However, in some special cases there is increased sensitivity to parameter variations. These special cases deserve the most careful analysis, since they are the ones that cause the most trouble when implementing optimal systems.

Problem definition

We will consider a continuous one-dimensional system that has the properties of controllability and observability:

$$\dot{x}(t) = A_c x(t) + b_c u(t);\quad y(t) = c_c x(t), \tag{1}$$

where A_c, b_c and c_c – $n\times n$, $n\times 1$ and $1\times n$ – matrices, respectively.

We discretize system (1) using the quantization period T and a zero-order extrapolator [6].

Assuming $x_k = x[kT]$, $y_k = y[kT]$, we will have

$$x_{k+1} = Ax_k + bu_k, \tag{2}$$

$$y_k = cx_k, \tag{3}$$

where

$$A = \exp(A_c T),\ b = \int_0^T \exp(A_c p)dp \cdot b_c \quad c = c_c. \tag{4}$$

In this case, the pair (A, b) is controllable, and the pair (c, A) is observable for almost all T. Let us apply the following control law ($k \geq n$-1) to system (2) - (4):

$$u_k = -g_1 u_{k-n+1} - g_2 u_{k-n+2} - \ldots - g_{n-1} u_{k-1} - h_1 y_{k-n+1} - h_2 y_{k-n+2} - \ldots - h_n y_k. \tag{5}$$

Let us rewrite (5), considering $u_0,\ldots,u_{n-2}$ as arbitrary initial values, for $k \geq 0$:

$$u_{k+n-1} = -g_1 u_k - g_2 u_{k+1} - \ldots - g_{n-1} u_{k+n-2} - h_1 y_k - h_2 y_{k+1} - \ldots - h_n y_{k+n-1}.$$

Then, taking into account (2) we will have

$$\begin{bmatrix} x_{k+1} \\ u_{k+1} \\ u_{k+2} \\ \vdots \\ u_{k+n-1} \end{bmatrix} = \begin{bmatrix} A & b & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \\ -\delta & -v_1 & \cdots & & -v_{n-1} \end{bmatrix} \begin{bmatrix} x_k \\ u_k \\ u_{k+1} \\ \vdots \\ u_{k+n-2} \end{bmatrix},$$

where

$$\begin{cases} \delta = (h_1, h_2, \ldots, h_n)\left(c^T, A^T c^T, \ldots, A_{n-1}^T c^T\right)^T, \\ v_1 = g_1 + h_2 cb + h_3 cAb + \ldots + h_n cA_{n-2} b, \\ v_2 = g_2 + h_3 cb + h_4 cAb + \ldots + h_n cA_{n-3} b, \\ \vdots \\ v_{n-1} = g_{n-1} + h_n cb. \end{cases} \tag{6}$$

Designating $X_k = (x_k^T, u_k, \ldots, u_{k+n-2})^T$ and $w_k = u_{k+n-1}$ we will consider

$$w_k = -(\delta, v_1, v_2, \ldots, v_{n-1}) X_k$$

as state feedback for an open-loop system [2, 7]:

$$X_{k+1} = \begin{bmatrix} A & b & & \\ & & I_{n-2} & \\ 0 & 0 & \cdots & 0 \end{bmatrix} X_1 + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} w_k. \tag{7}$$

Using a vector $f(1\times n)$ of feedback gains that stabilizes an ideal closed-loop system

$$x_{k+1} = (A - bf)x_k$$

we can choose k and v_i according to the relations

$$\delta = fA_{n-1},\ v_1 = fA_{n-2}b, \ldots, v_{n-2} = fAb,\ v_{n-1} = fb.$$

We will assume that for system (7) the $i = 0$ values at $u_k,\ldots,u_{k+n-2}$ are initial values, and at $k \geq 0$ the values $w_k = u_{k+n-1}$, are successive delays. The amplitude of the response to the input influence is determined by the value w_k. Let us assume that the cost indicator, which is minimized using w_k, has the form

$$J_y = \sum_{k=0}^{\infty}\left(x_k^T Q x_k + r w_k^2\right). \tag{8}$$

It can be shown that

$$f = \left(r + b^T P b\right)^{-1} b^T P A, \quad (9)$$

$$P = Q + A^T P A - A^T P b\left(r + b^T P b\right)^{-1} b^T P A, \quad (10)$$

and we have an optimum of indicator (8) if f in (9) is the vector of the optimal feedback gain [7], minimizing the indicator

$$J_0 = \sum_{i=0}^{\infty}\left(x_k^T Q x_k + r u_k^2\right)$$

for system (2).

Options $h_1, h_2, ..., h_n$ control law (5) are uniquely determined by relations (10) and (6), after which the parameters can be calculated $g_1, g_2, ..., g_{n-1}$

$$\left(h_1, h_2, ..., h_n\right) = f A_{n-1}\left(c^T, A^T c^T, ..., A_{n-1}^T c^T\right)^{-T},$$

$$\begin{cases} g_1 = f A_{n-2} b - h_2 cb - ... - h_n c A_{n-2} b, \\ g_2 = f A_{n-3} b - h_3 cb - ... - h_n c A_{n-3} b, \\ \vdots \\ g_{n-1} = fb - h_n cb. \end{cases}$$

Let us express equality (5) in the form of a pulse transition function:

$$\frac{U(z)}{Y(z)} = \frac{h_n z^{n-1} + h_{n-1} z^{n-2} + ... + h_2 z + h_1}{z^{n-1} + g_{n-1} z^{n-2} + ... + g_2 z + g_1}. \quad (11)$$

Expression (11) is the equation for the optimal compensator.

If we take the delay into account, it is obvious that u_k, cannot be obtained from y_k. However, under the assumption that $u_0, ..., u_n$ are arbitrary initial values, relation (5) for $k > 0$, takes the form

$$u_{k+n} = -g_1 u_k - ... - g_n u_{k+n-1} - h_1 y_k ... - h_n u_{k+n-1}. \quad (12)$$

Therefore, similar results can be obtained in this case. However, although the quantities g_k can be determined uniquely, system (11) is not necessarily stable [7]. Let us define for $0 < m < 1$ the quantity

$$\hat{y}_k = y[kT + mT] \text{ and } \hat{x}_k = x[kT + mT].$$

Let $u_0, ..., u_{n-1}$ – arbitrary initial values. Then the implemented control law can be constructed for $k \geq 0$ as follows [4, 7]:

$$\begin{aligned} u_{k+n} = -g_1 u_i - g_2 u_{k+1} - ... - g_n u_{k+n-1} - h_1 y_k - p_1 \tilde{y}_k - \\ - p_2 y_{k+1} - p_2 \tilde{y}_{k+1} - ... - h_n y_{k+n-1} - p_n \tilde{y}_{k+n-1}. \end{aligned} \quad (13)$$

Relationship (13) coincides with relationship (12) with the exception of additional terms $p_1 \tilde{y}_k$. Let us denote by $T_0 = T - mT$ the computation time. Then at $t = kT + mT$

$$\tilde{x}_k = \tilde{A} x_k + \tilde{b} u_k,$$
$$\tilde{y}_k = c \tilde{x}_k,$$

where

$$\tilde{A} = \exp\left(A_c mT\right), \; \tilde{b} = \int_0^{mT} \exp\left(A_c p\right) dp \cdot b_c, \quad c = c_c. \quad (14)$$

Substituting (14) and (2)-(4) into (13), we get

$$\begin{aligned} u_{k+n} &= \left(g_1 - p_1 c\tilde{b} + h_2 cb + p_2 c\tilde{A}b + ... + h_n c A^{n-2} b + p_n c\tilde{A} A^{n-2} b\right) u_k - \\ &- \left(g_2 - p_2 c\tilde{b} + h_3 cb + p_3 c\tilde{A}b + ... + h_n c A^{n-3} b + p_n c\tilde{A} A^{n-3} b\right) u_{k+1} - ... - \left(g_n - p_n c\tilde{b}\right)_{k+n-1} - \\ &- \left(h_1, h_2, ..., h_n, p_1, p_2, ..., p_n\right)\left(c', A'c', ..., A'^{n-1} c', ..., \tilde{A}'c', \tilde{A}'A'c', ..., \tilde{A}'^{n-1} c'\right)' x_k = \\ &= -\delta x_k - \upsilon_k u_k - \upsilon_2 u_{k+1} - ... - \upsilon_n u_{k+n-1}. \end{aligned} \quad (15)$$

Let us consider them $g_1, ..., g_n$ as free parameters and choose these parameters so that the polynomial

$$z^n + g_n z^{n-1} + ... + g_2 z + g_1$$

was stable. If δ, $v_1, ..., v_n$ are defined, then comparing different parts of relation (15), we determine $h_1, ..., h_n$, $p_1, ..., p_n$ from the following equation:

$$(h_1, ..., h_n, p_1, ..., p_n) M = (k, v_1 - g_1, ..., v_n - g_n), \quad (16)$$

where

$$M = \left[\begin{array}{c|ccccc} c & 0 & & & & 0 \\ cA & cb & 0 & & & \vdots \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ cA^{n-1} & cA^{n-2} b & \cdots & \cdots & cb \; \cdots & 0 \\ \hline c\tilde{A} & c\tilde{b} & 0 & & \cdots & 0 \\ c\tilde{A}A & c\tilde{A}b & c\tilde{b} & \cdots & & \vdots \\ c\tilde{A}A^2 & c\tilde{A}Ab & & \ddots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 \\ c\tilde{A}A^{n-1} & c\tilde{A}A^{n-2} b & c\tilde{A}A^{n-3} b & \cdots & c\tilde{A}b \; \cdots & c\tilde{b} \end{array}\right]. \quad (17)$$

We rewrite equation (16) in the following form:

$$zM = l, \quad (18)$$

where $z = (h_1, ..., h_n, p_1, ..., p_n)$ and $l = (\delta, v_1 - g_1, ..., v_n - g_n)$ are dimension vectors ($1 \times 2n$).

When finding a solution to equation (18), computational difficulties arise due to the fact that the system of equations (18) when finding z may be poorly conditioned. This means that when solving equation (18), it is necessary to use regularization methods [8-11].

Solution of the task

To problem (18) in its general formulation we apply the method of A.N. Tikhonov [10-12]. The simplest option of A.N. Tikhonov is to find the minimum point of the functional $\Phi_\alpha(z) = \alpha\|z\|_H^2 + \|zM - l\|_F^2$, $z \in H$, where α is a small positive parameter. This minimum problem is equivalent to determining z from the equation

$$\alpha z + zMM^T = lM^T \qquad (19)$$

Along with the described version of the method by A.N. Tikhonov (19), one can use its iterated version. Let's set the initial approximation $z_{0,\alpha} = z_0 \in H$ and a natural number $2n \geq 1$ and sequentially calculate $z_{1,\alpha},\ldots, z_{2n,\alpha}$ using the formulas [12-14]:

$$\alpha z_{m,\alpha} + z_{m,\alpha}MM^T = \alpha z_{m-1,\alpha} + lM^T \quad (m = 1,\ldots,2n); \qquad (20)$$

element $z_{m,\alpha}$ will be taken as an approximate solution to equation (18).

The solution to equation (20) will be determined by the formula

$$z_\alpha = lM^T(MM^T + \alpha I)^{-1} = lM^T g_\alpha(MM^T),$$

and the solution to equation (20)

$$z_{2n,\alpha} = (I - MM^T g_{2n,\alpha}(MM^T))z_0 + lM^T g_{2n,\alpha}(MM^T),$$

where $g_\alpha(\lambda)$ and $g_{2n,\alpha}(\lambda)$ – are written in the form

$$g_\alpha(\lambda) = (\alpha + \lambda)^{-1}, \quad 0 \leq \lambda < \infty.$$

$$g_{2n,\alpha}(\lambda) = \sum_{j=0}^{2n-1} \frac{\alpha^j}{(\alpha+\lambda)^{j+1}} = \frac{1}{\lambda}\left[1 - \left(\frac{\alpha}{\alpha+\lambda}\right)^{2n}\right], \qquad 0 \leq \lambda < \infty$$

As above, we will define $X_k = \left(x_k^T, u_k,\ldots, u_{k+n-1}\right)^T$, and relation (15) will be considered as state feedback for an open-loop system

$$X_{k+1} = \begin{bmatrix} A & b & & \\ & & & I_{n-1} \\ & & & \\ 0 & 0 & ,..., & 0 \end{bmatrix} X_k + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u_{k+n}. \qquad (21)$$

Believing

$$\delta = fA^n,\ v_1 = fA^{n-1}b,\ldots, v_{n-1} = fAb,\ v_n = fb$$

and choosing f so that $(A - bf)^n = 0$, we obtain a closed system (21), (15), which is reset to zero in $2n$ quantization periods.

Conclusion

If f is selected according to (9), (10), optimal control is realizable if w_k, in relation (8) is replaced by u_{k+n}.

In the case when the pair (c_c, A_c) is observable and $det[A_c] \neq 0$, if $c_c A_c^{-1} b_c \neq 0$ (that is, system (1) has no poles and zeros at the origin), then for any given $T \neq 0$ matrix $M(2n \times 2n)$ of the form (17) is nonsingular for almost all m, $0 < m < 1$.

Thus, the $h_1,\ldots,h_n$ quantities $p_1,\ldots,p_n$ are determined uniquely from relation (16).

The above algorithms make it possible to synthesize a stable control system with an optimal feedback gain.

References

1. Alexandrov A.G.: Optimal and adaptive systems. Moscow, Higher School, 1989.
2. Khlebnikov M.V., Shcherbakov P.S. “Synthesis of optimal feedback under limited control”, J. Avtomat. and Telemekh., 2014, No. 2, –PP. 177-192.
3. Matveev A.S., Yakubovich V.A.: Optimal control systems: Ordinary differential equations. Special tasks. Publishing house: St. Petersburg University, 2003. 540 p.
4. Vanko V.I. and others. Calculus of variations and optimal control. Moscow, Publishing house of MSTU im. Bauman, 2006. 487 p.
5. Afanasyev V.N.: Theory of optimal control of continuous dynamic systems. Moscow, Publishing House of the Faculty of Physics of Moscow State University, 2011. 168 p.
6. Egupov N.D., Pupkov K.A. Methods of classical and modern theory of automatic control. Textbook in 5 volumes. Moscow, Publishing house of MSTU named after N.E. Bauman, 2004.
7. Mita, T., Pang, B.C., & Liu, K.Z. “Design of optimal strongly stable digital control systems and application to output feedback control of mechanical systems”, International Journal of Control, 45(6), 2071-2082. 1987. https://doi.org/10.1080/00207178708933868
8. Igamberdiev Kh.Z., Yusupbekov A.N., Zaripov O.O.: Regular methods for assessing and managing dynamic objects under conditions of uncertainty. Tashkent, Tashkent State Technical University, 2012. 320 p.
9. Igamberdiev Kh.Z., Sevinov J.U., Zaripov O.O.: Regular methods and algorithms for the synthesis of adaptive control systems with customizable models. Tashkent, Tashkent State Technical University, 2014.
10. Tikhonov A.N., Arsenin V.Ya.: Methods for solving ill-posed problems, Moscow, Nauka, 1986. 288 p.
11. Tikhonov A.N., Goncharsky A.V., Stepanov V.V., Yagola A.G.: Numerical methods for solving ill-posed problems, Moscow, Nauka, 1990.
12. Tikhonov A.N., Goncharsky A.V.: Ill-posed problems in natural science. Moscow, Moscow University Publishing House, 1987. 299 p.
13. Mamirov U.F.: Regular synthesis of adaptive control systems for uncertain dynamic objects. Tashkent, Knowledge and intellectual potential, 2021. 215 p.
14. Mamirov U.F. “Sustainable Algorithms For Synthesis Of Regulators In Adaptive Control Systems Of Parametrically Uncertain Objects”, J. Chemical Technology, Control and Management, 2019 (4), PP. 126-132. https://doi.org/10.34920/2019.3.126-132