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DEVELOPMENT OF ALGORITHM OF OPTIMUM CONTROL BY MULTIDIMEN-SIONAL DISCRETE DYNAMIC OBJECTS BY QUICK CRITERIA

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The article deals with the development of a synthesis algorithm in the sense of the optimal control of multidimensional dynamic objects by the criterion of speed. To solve this problem, we propose a modified method based on the representation of a discrete system in the form of a space of state variables and using the theorem on N-intervals. When determining the values of control actions, it is proposed to use the values of predicted errors that allow improving the quality of the control process and will provide high accuracy of calculation of control actions.

Key words: *pulse-width modulator, optimal control synthesis of methods and algorithms, discretization of dynamic models, microprocessor controller, differential equations.*

A lot of works, methods and algorithms have been devoted to solving the problem of synthesizing optimal control actions in systems with linear modulation of signals [1-3]. The main disadvantages of these methods are their extremely cumbersome and complex mathematical apparatus, a large number of simplifying sentences and calculations, the complexity of the interpretation of the results. In addition, the use of these methods often leads to systems of partial differential equations or algebraic transcendental equations, the exact solution of which is impossible; in case of incorrectness of individual calculations, there may in principle not be a solution; the use of numerical methods of solution with a large dimensionality of the resulting system, even when using the capabilities of modern computers, can give an absolutely unacceptable result. In addition, the use of existing methods for solving such problems, even using modern computer technology, cannot guarantee acceptable results. Therefore, the development of machine-oriented methods for the synthesis of control actions in systems with pulse-width modulation applications that do not require special mathematical training is an urgent task. To solve this problem, a method is proposed based on the interpretation of the dynamics of impulse systems in the form of a space of variable states and the use of the theorem on N-intervals [4]. Using this method for systems with non-linear modulation of the control action requires a modification of the known algorithm for solving the problem of translating a multidimensional linear dynamic object that has M input and N output controlled variables from a given initial state to the desired final state for a minimum number of control cycles. It is assumed that the sampling period for all input signals is the same. The minimum possible number of clock ticks in accordance with the theorem on N - intervals is determined by the expression:

$$L = Int\left\{\sum_{i=1}^{N}\sum_{j=1}^{M} P_{ij} / M + 0.5\right\}. \quad (1)$$

where is P_{ij} - the order of the transfer function (differential equation) of the channel; *j*-th input; *i*-th output of the control object.

The required state of the control object is determined by the conditions:

$$Y_i(L+K) = G_i(L+K),\ \mathrm{i} = \overline{1,\mathrm{N}}, \quad \mathrm{K} = \overline{0,\tilde{\mathrm{N}}_i}, \ (2)$$

where is $Y_i(L+K)$ - the value of the *i*-th output variable in the $(L+K)$ - m step

$G_i(L+K)$-Required value of the i-th output variable; C_i - the number of clock cycles of fixing the *i*-th output variable.

Based on the analysis of the dynamics of the behavior of the control object, we change the conditions (2) to the following

$$Y_i(L+K)=E_i(L+K),\ \mathrm{i}=\overline{1,\mathrm{N}},\ \ \mathrm{K}=\overline{0,\tilde{\mathrm{N}}_\mathrm{i}}\ .\quad (3)$$

where $E_i(L+K)=G_i(L+K)-Y_i^*(L+K)$, (4)

$Y_i^*(L+K)$ - The predicted value of the i-th output variable, provided:

$$U_j(m)=0,\ \mathrm{j}=\overline{1,\mathrm{M}},\quad \mathrm{m}=\overline{1,\mathrm{L}}.\qquad (5)$$

Based on dynamic models, it is easy to obtain dependencies connecting the output variables of an object with its input variables for linear impulse systems. In the case of zero initial conditions, these dependencies will have the form:

$$Y_i(L+K)=\sum_{j=1}^{M}\sum_{m=1}^{L}U_j(m)*\omega_{ij}((L+K-m+1)*T),\ \ \mathrm{i}=\overline{1,\mathrm{N}},\quad \mathrm{K}=\overline{0,\mathrm{C}_\mathrm{i}},\quad (6)$$

Where is T -the sampling period of the control signal; $\omega(qT)$ is the value of the weight function (reaction to an impulse of duration T) in the q-th step.

Combining the system of expressions (6) with conditions (3), we obtain the system of linear algebraic equations:

$$\bar{W}*\bar{U}=\bar{E},\qquad (7)$$

where is W -the coefficient matrix of weight functions

$$W=\lfloor\omega_{ij}(L+K-m+1\rfloor\qquad (8)$$

U - Column vector of the predicted error values:

$$\mathrm{U}=[\mathrm{U}_1(1),\mathrm{U}_1(2),......,\mathrm{U}_1(\mathrm{L}),\mathrm{U}_2(1),.....,\mathrm{U}_2(\mathrm{L}),\mathrm{U}_\mathrm{m}(1),....,\mathrm{U}_\mathrm{m}(\mathrm{L})]^\mathrm{T}\qquad (9)$$

E-vector column of predicted error values:

$$\mathrm{\AA}=[\mathrm{\AA}_1(1),\mathrm{\AA}_1(2),......,\mathrm{\AA}_1(\mathrm{L}),\mathrm{\AA}_2(1),.....,\mathrm{\AA}_2(\tilde{\mathrm{N}}_2+\mathrm{L}),\mathrm{\AA}_\mathrm{n}(\mathrm{L}),....,\mathrm{E}_\mathrm{n}(\mathrm{C}_\mathrm{i}+\mathrm{L})]^\mathrm{T}\qquad (10)$$

The dimension of system (7) is equal to: $M*L=\sum_{i=1}^{N}C_i$ (11)

Having solved the system of linear equations (7), we obtain the desired control actions in the form of linear combinations of predicted errors:

$$\mathrm{U}_\mathrm{j}(\mathrm{m})=\sum_{\mathrm{i=1}}^{\mathrm{N}}\sum_{\mathrm{K=0}}^{\mathrm{C}_1}\mathrm{R}_{\mathrm{im}}\left(\sum_{\mathrm{s=1}}^{\mathrm{i=1}}\mathrm{C}_\mathrm{s}+\mathrm{K}\right)*\mathrm{E}_\mathrm{i}(\mathrm{L}+\mathrm{K}),\qquad (12)$$

where is R_{im} - vector - row of the matrix ω^{-1}.

Substituting into the expression (12) the values of the predicted errors in accordance with the initial conditions that are currently changing, we can calculate the numerical values of the control actions.

The resulting expression (12) is actually the main corrective procedure in the iterative search for control actions modulated in width. But first, consider some necessary conditions, the implementation of which should provide a solution to the task. Firstly, the pulse repetition period for pulse-width modulation should be equal to the sampling period of the control signal during synthesis for a linear pulse system. Secondly, the condition must be met:

$$|U_j(m)|<A_j,\qquad (13)$$

where is A_j - the amplitude of the modulated width control actions.

If the failure of the first condition is trivial, then the 2nd condition, as a rule, requires considerable laboriousness in the calculation. To overcome this difficulty, we consider one of the approaches ensuring the fulfillment of condition (13), which consists in artificially increasing the number of control cycles.

Suppose that the solution of the synthesis problem for a linear pulse system with the number of translation clocks defined by expression (1) led to the failure of condition (13).

Increase the number of clock ticks by J. First, take $J = 1$. Then the values of the predicted errors will change, and will be determined by the expression:

$$E_i^j(L+K) = E_i(L+K) - \sum_{j=1}^{M}\sum_{m=1}^{J} U_j(m) * \omega_{ij}(L+K-m+1), \quad (14)$$

$$i = \overline{1,N}, \quad K = \overline{1;C_i+J},$$

Where we substitute the found expressions for the predicted errors into expressions (12), which will allow us to express L as the main control actions for each input variable J through additional controls:

$$U_j(J+m) = U_j(j+m) + \sum_{K=1}^{M}\sum_{i=1}^{J} \omega_{ij}(L+k-m+1+J) * U_k(i) \quad (15)$$

Now it is necessary to solve the optimization problem associated with minimizing the criterion:

$$F = \sum_{j=1}^{M}\sum_{i=1}^{L+J} U_j^2(i) \to \min, \quad (j = \overline{1,M};\ i = \overline{1,J}) \quad (16)$$

The above problem is solved simply by using the least squares method. As a result of its solution, the values of auxiliary control actions are found $U_j(k), (k = 1, J)$. If all of them satisfy condition (13), then using formula (14) we find the values of the predicted errors, substitute them into expression (12) and find the values of the control actions $U_j(k), (k = \overline{J+1, L+J})$.

They also need to be checked for the fulfillment of condition (13). If it is, then you can go to the next step in the synthesis. Otherwise, it is necessary to increase the value of J by one and repeat the procedure of minimizing the sum of the squares of the control actions.

After we achieve condition (13) because of a gradual increase in the number of control clocks, we proceed directly to the iterative procedure for the synthesis of control actions in the class of pulse-width signals. The solution to this problem is based on the position that the total area of control pulses for each output for a linear pulse system and a system with pulse-width control modulation should be equal. In this case, the values of the control actions found during the synthesis for a linear pulse system are corrections in the form of a change in the area of the corresponding control signals modulated in width. Define the main stages of solving the problem:

1) Accept $\tau_i(i) = /U_i(m)/$,

$$v(i) = \begin{cases} A_j^* \sin g\{U_j(i)\}, (i-1)*T \prec t \le in(i=1)*T + \tau_j(i); \\ 0, in(i-1)*T + \tau_j(i) \prec t \le i*T, \end{cases}$$

where $i = \overline{1,M}; i = \overline{1,J}$. (17)

The above control actions are auxiliary, focused on minimizing the amplitudes of the control actions, and therefore do not change further.

2) The state of the system at the time $t = JT$ is determined. This state is taken as the initial one, remains unchanged in the future, and serves to calculate the predicted errors at subsequent times.

3) The iteration number is determined:

$q = 0;\ U_j^q(i) = 0;\ \tau_j^q(i) = 0;$ Where $j = \overline{1,M};\ i = \overline{J+1, J+L}$. (18)

4) The values of the transient in the system are calculated for:

5) $t = J + L + K; K = \overline{0, C_i}$.

6) Predicted error values are determined $E_i^q(J+L+K);\quad i = \overline{1,N};\ K = \overline{0,C_i}$.

7) According to formulas (12), control actions for a linear pulse system are calculated. It is assumed that the coefficients R_{jm} are determined previously. Denote the found values of the effects in the form $Uj(J+K)$.

8) We calculate analogues of control actions for a pulse-width system:

$U_j^{q+1}(i) = U_j^q(i) + U_j(i)$, $\tau_j^{q+1}(i) = /U_j^{q+1}(i)/A_j; j=\overline{1,M}; i=\overline{J+1,J+L}$.

Using the formula (18), we determine the type of control action.

9) if $/U_j(i)/ < \varepsilon; j=\overline{1,M}; i=\overline{J+1,J+L}$

where is ε - the predetermined calculation error in advance, therefore, a solution is found and then proceeds to step 4.

Consider the application of the proposed methodology as an example of the synthesis of control actions for a system having the structure shown in Fig. 1.

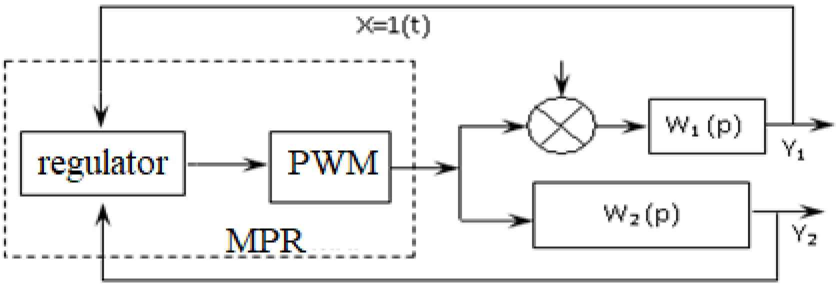

Fig. 1. The structure of the pulse width system

let be $W_1(p) = 1/5p+1; W_2(p) = 5/2(2p+1); T=1; G_1=5; A=100; G_2=2$

we find $h_1(t) = L^{-1}[W_2(p)/p] = 5(-2+t+2e^{-t/2})$ we calculate the values of transients and weight functions:

In order for the steady-state value to be $Y_1(t)$ equal $G_1=5$, we take $X=5$. For this system we have $L=3; C_1=0; C_2=1$.: From the coefficients of the weight functions, we form a system of linear equations of the form (7):

$$W = \begin{bmatrix} 0.12152 & 0.14841 & 0,18127 \\ 3.5525 & 2.6135 & 1,0653 \\ 4.12205 & 3.5525 & 2,6135 \end{bmatrix}$$

The solution of the system of linear equations (7) gives the following result:

$$\left.\begin{aligned} U(1) &= -14,3*E_1(3) - 12,07*E_2(3) + 14,88*E_2(4) + 64,8*X \\ U(1) &= -23,07*E_1(3) + 20,254*E_2(3) - 24,2564*E_2(4) - 104,088*X \\ U(3) &= -87,09*E_1(3) - 8,49*E_2(3) + 9,88*E_2(4) + 39,295*X \end{aligned}\right\} \quad (19)$$

Taking the initial conditions as zero, we get:

$$E_1(3) = G_1;\ E_2(3) = G_2;\ E_2(4) = G_2.$$

Substitution of the obtained values in (19) gives:

$$U(1) = -388,42; \qquad U(2) = -625,014; \qquad U(3) = -236,186;$$

It is easy to verify that condition (13) is not satisfied. We accept J = 1, then:

$$E_1(4) = G_1 - \omega_1(4)*U(1)$$
$$E_2(4) = G_2 - \omega_2(4)*U(1);$$
$$E_2(5) = G_2 - \omega_2(5)*U(1)$$

Substitution of these expressions in (21) instead of E1 (3), E2 (3), E2 (4) gives:

$U(2) = -388{,}42 - 2{,}43 * U(1)$

$U(3) = 625{,}014 + 1{,}93 * U(1)$;

$U(4) = -236{,}186 - 0{,}5 * U(1)$

We use the usual least-squares procedure to minimize the function:

$F = \sum_{k=1}^{4} U^2(k) \to \min$, $\frac{\partial F}{\partial U(1)} = 4537{,}47 + 21{,}77 * U(1) = 0$. Where do we find $U(1)$ = -208.44

Since the control found does not satisfy condition (13), we accept $J = J + 1$ and perform the minimization of the function with respect to the variables $U(1)$ and $U(2)$:

$$F = \sum_{k=1}^{5} U^2(k) \to \min$$

Because of the calculation, we obtain $U(1)$ = -135.6 and $U(2)$ = 21.5. Since condition (13) is not satisfied, we take $J = J + 1$=3 and minimize the criterion with respect $F = \sum_{k=1}^{6} U^2(k) \to \min$ to the variables, $U(1)$, $U(2)$, $U(3)$. As a result, we get:

$U(1)$=-86.273; $U(2)$=-20.73; $U(3)$=37.6; $U(4)$=73.02; $U(5)$=56.77; $U(6)$=59.747

All the values obtained above satisfy (13), therefore, we can proceed to the iterative procedure for searching for modulated width effects.

Accept $\tau_1 = 0{,}862773$; $\tau_2 = 0{,}2073$; $\tau_3 = 0{,}3736$;

$$V_1 = \begin{cases} -100, & 0 \le t \le \tau_1 \\ 0, & \tau_1 < t \le 1 \end{cases} \quad V_2 = \begin{cases} -100, & 1 \le t \le 1.2073 \\ 0, & 1.2073 < t \le 2 \end{cases} \quad V_3 = \begin{cases} -100, & 2 < t \le 2.3736 \\ 0, & 2..3736 < t \le 3 \end{cases}$$

It is necessary to find, τ_4, τ_5, τ_6, and, accordingly, V4, V5, V6,

The results of an iterative search in accordance with the above methodology are given in table 1:

Table 1.

Values of control actions and output signals according to the results of iterative search

G	U(4)	U(5)	U(6)	4	5	6
0	76,1575	50,48	-56,6	0,7616	0,5048	0,566
1	-29,428	48,97	-19,544	0,4359	1	0,7355
2	7,616	-2,297	-5,3	0,512	0,977	0,79
3	0,03672	-2,834	2,798	0,5124	0,9487	0,79
4	-0,948	1,35	-0,402	-0,402	0,503	0,76
5	0,36	-0,199	-0,16	0,50695	0,9602	0,7647
6	-0,00986	-0,1157	0,1256	0,50645	0,959	0,765
7	-0,051	0,08	-0,029	0,5059	0,96	0,768

The found control actions allow reducing the error of regulation of dynamic objects by 3%. If necessary, regulation errors can be minimized, by further iterative search.

In conclusion, we can say that the developed synthesis algorithm is optimal by the criterion for the speed of discrete control systems for multidimensional dynamic objects. The proposed modified method, based on the representation of a discrete system as a space of variable states and the use of the N-interval theorem, makes it possible to obtain accurate calculations, including those based on economic criteria. A comparative analysis of the results obtained with known methods showed that the application of the proposed method allows to increase the accuracy of the calculation of control actions by 10% compared to existing methods and to ensure a reduction in regulation error by 3%.

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